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A NOTE ON NON-NEWTONIAN LAMINAR/TURBULENT TRANSITION

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Abstract: In this work a simple new approach to laminar/turbulent transition is explored, which is not seen as an advance on the current theory, but instead as an avenue for possible future development or refinement. A new universal empirical model is presented for the prediction of friction factors for non-Newtonian fluids in the laminar/turbulent transition region, covering the previously unpredictable middle ground between the laminar region (where the Hagen-Poiseuille equation applies), and the turbulent region (where the Colebrook-White equation or Blasius equation are typically applied). The new model attempts to predict the "dip" below the turbulent model curves that is exhibited differently by non-Newtonian fluids of differing viscous characteristics. It is typically observed that fluids of relatively low viscosities, such as water, jump quite suddenly from laminar behavior to turbulent behavior, as opposed to concentrated slurries, which appear to have smoother transitions that are spread out over a larger range of Reynold's numbers. The new model has been calibrated with some 800 points of experimentally measured non-Newtonian open channel data from the Haldenwang (2003) data set, and is presented here for the wider community to test and validate with other data.

KEY WORDS: Transition, turbulent, laminar, non-Newtonian.

NOTATION

f_L	Fanning friction factor (equal to one quarter of the Darcy friction factor)
f_{LV}	Fanning friction factor co-ordinate of the predicted transition curve vertex
K	Herschel-Bulkley rheological model consistency index
k_s	Roughness of the channel (m)
n	Herschel-Bulkley rheological model flow index
Re_V	Reynolds number co-ordinate of the predicted transition curve vertex
R_H	Hydraulic Radius of a channel (m)
η_B	Bingham plastic viscosity of the slurry, with the Bingham model tangent imposed at a shear rate of at least 400 s^{-1}
τ_y	Yield stress of the fluid (Pa)

1. INTRODUCTION

In the design of tailings slurry transport systems (pipelines and open channels), as well as in the prediction of tailings beach slopes, the slurry is often flowing in the transition turbulent regime. Presently the prediction of head losses in this transition regime remains a mystery to the designers of tailings handling and storage infrastructure, and indeed also to the fluid dynamics specialists and the much wider community of engineers applying fluid dynamics to a range of other design situations. In fluid dynamics text books a Moody diagram will typically be found with a shaded region covering the unknown region of transition, and the accompanying text will typically contain statements such as "the transition region is found to be unstable and unpredictable" (Schaschke, 1998). Some workers have stated that a fluid flowing in the transition region experiences random switching between laminar and turbulent flow conditions (Slatter, 2013). Others have suggested that transitional flows are laminar near the stationary boundaries and turbulent in the middle of the cross section (White, 2011). As a result, many designers extend the established empirical turbulent flow models (such as the Blasius equation or the Colebrook-White equation) into the transition region, even though it is found that the friction factor generally dips below these turbulent models in the transition region.

For Newtonian fluids in pipe flow, the friction losses make a sudden jump from the laminar regime to the turbulent regime, with a very narrow transition zone between them. This sudden jump is illustrated in Figure 1, with a plot of the gas data of McKeon et al (2004) and the water data of Nikuradse (1932) on a Moody diagram.

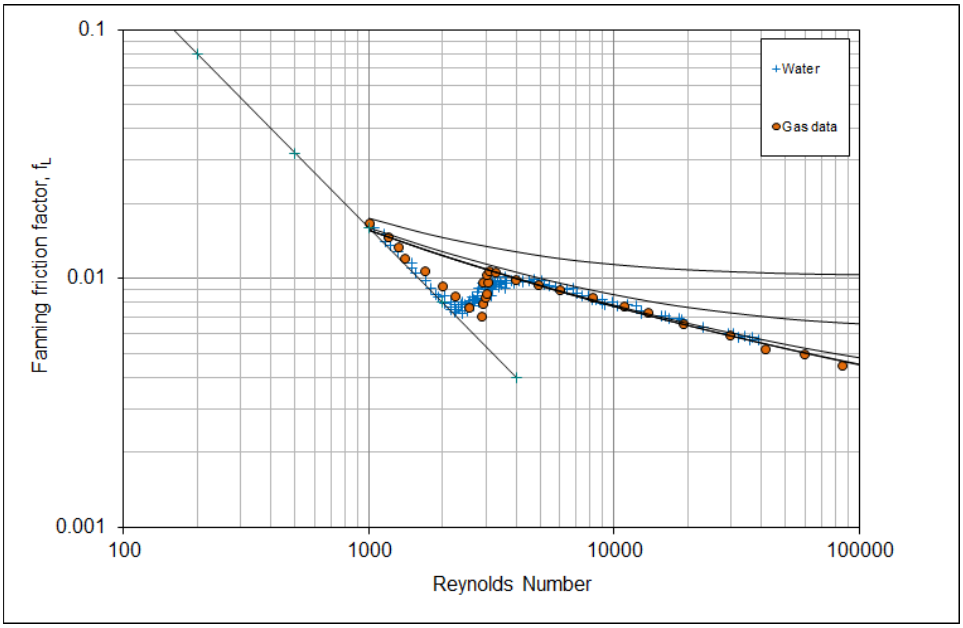

Figure 1 – Moody diagram featuring the Newtonian data from the McKeon et al (2004) and Nikuradse (1932) data sets, illustrating the transition behaviour of a gas and water.

Nikuradse (1932) presented an empirical equation for predicting the transition data in his data set for water. Cheng (2008) also presented a model to fit the water data of Nikuradse (1932), whilst Joseph and Yang (2009) presented a five-part connected model (with each part covering a specific range of Reynolds numbers), based on an empirical fit to the McKeon et al (2004) data set for gases and the Nikuradse (1932) data set for water.

It is noted that the gas data presented in Figure 1 makes a dramatic jump over a very limited range of Reynolds numbers (about 2900 to 3100), whilst the transition jump for the water data covers a Reynolds number range from about 2100 to about 5000. With non-Newtonian fluids a more prolonged transitional "dip" can be observed instead of a jump, with friction losses typically converging in a seemingly random way towards a smooth curve defining turbulent conditions. This "dip" behavior has been well illustrated in the Moody diagram presented by Metzner and Reed (1955), presented as Figure 2.

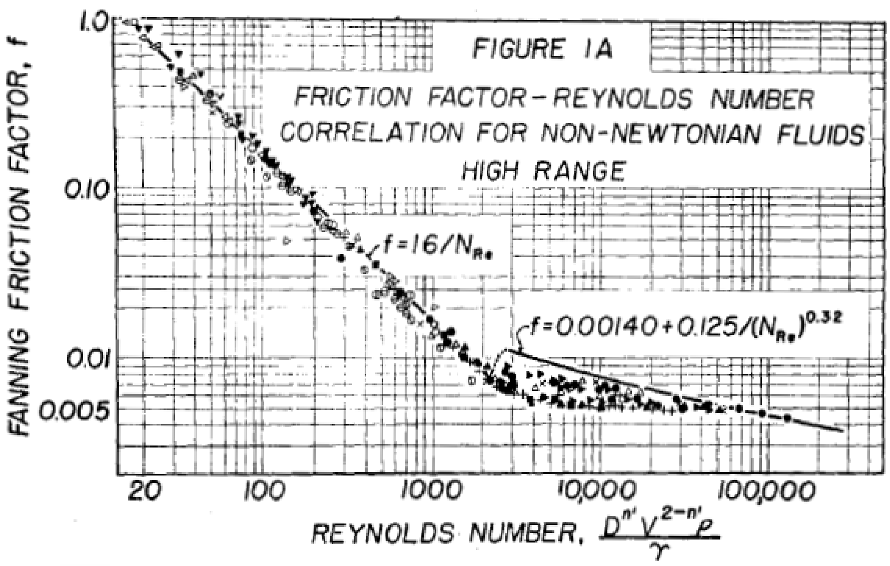

Figure 2. Moody diagram presented by Metzner and Reed (1955), which shows the transitional data dipping below the turbulent model curve.

In Figure 2, all of the transitional experimental data plotted by Metzner and Reed (1955) fell below the turbulent flow curve, with each data set exhibiting a "dip" in the transitional region. It is noted that the amount of dip can seem quite random from one fluid to the next, and that the onsets of transition and full turbulence can vary from one fluid to the next.

Haldenwang (2003) presented new empirical models for predicting the onset of transitional flow (from the laminar region), and the onset of fully turbulent flow (from the transition region), but he did not present any model for the prediction of the friction factor in the transition region. Other authors have presented methods for predicting the onset of turbulent flow in channels (Ryan and Johnson 1959) and fully turbulent flow in pipe flow (Slatter and Wasp 2000), (Wilson and Thomas 2006), but like Haldenwang, these workers did not present a model for predicting the friction factor in the transition region. It is believed that Metzner and co-workers' contributions are mainly associated with pseudo-plastic or power law rheological behavior as seen in Figure 2; i.e. fluids not exhibiting any yield stress. Haldenwang's (2003) data for transitional behavior comes from bentonite and kaolin products, which are characterized by a yield stress and a viscosity parameter.

2. OBJECTIVE AND SCOPE

The objective of this paper is to present a new universal empirical model that predicts the Fanning friction factor for non-Newtonian fluids flowing in the transition region, both in pipes and channels. This model is not presented as a development of the theoretical understanding of transition, but as a practical and simplistic approach that might hopefully aid designers. It is also hoped that the general approach might help others to develop a superior model in future.

The empirical model consists of two parts; the first part is an equation for a curve on the Moody diagram that follows the general "dip" behavior of a non-Newtonian fluid in the transition region. The second part is an equation that shifts that curve up, down, left or right to suit a given non-Newtonian fluid.

The model then applies the following approach: if the Hagen-Poiseuille equation predicts a friction factor that is smaller than that of the new model for a given Reynolds number, the new model overrides it. If the Colebrook-White equation predicts a friction factor for a given Reynolds number that is larger than the new model, the new model overrides it. This effectively enables the model to predict different transition ranges and friction factors for different fluids. The Hagen-Poiseuille equation remains the operative predictor for laminar flows, and the Colebrook-White remains for turbulent flows.

3. THE NEW MODEL

4.1 DEVELOPMENT OF THE MODEL

Some 792 points of experimental data from the Haldenwang (2003) data set have been used for calibrating the empirical model parameters. This data set consists of open channel data featuring various non-Newtonian fluids and channel cross sections.

The applicable friction factor for each data point was back-calculated using the Darcy-Weisbach equation. The Reynolds number for each point was calculated using the equation presented by Haldenwang et al (2004):

$$Re = 8\rho V^2/(\tau_y+K(2V/R_H)^n) \qquad (1)$$

It is noted that Equation 1 is presented for the Herschel-Bulkley rheological model, but it can also be applied to Power Law fluids (by making $\tau_y = 0$) or Bingham plastics (by making $n = 1$). Also, the equation can be applied to pipes by substituting D/4 for R_H.

While the Haldenwang (2003) data set provides an excellent amount of useful data, it was found that the trends in the data were not particularly strong, which made empirical modelling frustrating. Figure 3 illustrates this point with three different concentrations of kaolin slurry plotting in a seemingly random order. Furthermore, it was found that very few subsets in the Haldenwang data set reached full turbulence, with much of the data firmly in the laminar region. Such data was of limited value in calibrating a model for the transition region.

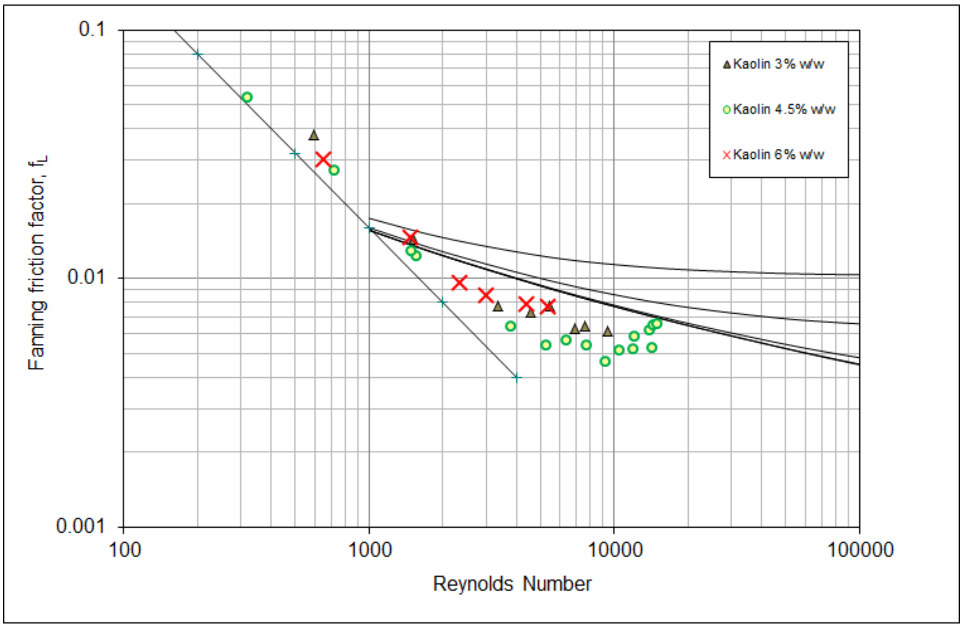

Figure 3. Moody diagram presenting a small selection of the Haldenwang (2003) data for three concentrations of Kaolin slurry. Note the apparently random order of the three sets.

Various trends were explored empirically on a trial and error basis, and it was found that the strongest trends were observed as a function of the Bingham plastic viscosity. It is noted that the Bingham plastic viscosity has been defined at a tangent of 400 s^{-1} from each rheogram.

The new transition model is presented as follows; for a non-Newtonian fluid of a nominal Bingham plastic viscosity, the Fanning friction factor in the transition region is calculated using the following equations:

$$f_L=f_{LV}(6(Re/((Re_V)/4900))^{-0.85}+1.5x10^{-7}(Re/((Re_V)/4900))^{1.17})/.0075 \quad (2)$$

Equation 2 defines the general transition curve that will be used for all predictions. This curve is meant to emulate the general shape of the "dip" that is observed in the experimental non-Newtonian data when it is plotted on a Moody diagram. Equations 3 and 4 define the location for the vertex of the transition curve, essentially causing the general transition curve to be translated vertically and horizontally to suit a fluid of a given plastic viscosity.

$$f_{LV} = -26.5\ \eta_B^{\,2}+1.1\ \eta_B +0.0022 \quad (3)$$

$$Re_V = 2.2\mathrm{x}10^7\ \eta_B^{\,2} - 9.0\mathrm{x}10^5\ \eta_B +9600 \quad (4)$$

Either side of the transition region, the Hagen-Poiseuille equation applies for the laminar region:

$$f_L = 16/Re \quad (5)$$

And the Colebrook-White equation applies for the turbulent region.

$$f_L=1/4\ (-2*LOG((k_s)/(14.8*R_H)+2.51/(Re*\ (4f_L)^{0.5})))^{-2} \quad (6)$$

It is noted that the Colebrook-White equation is implicit, but it can easily be iteratively applied in a spreadsheet to converge to 5 decimal places after about 6 iterations. A few logic statements in a spreadsheet enable the predictions from the three models (laminar, transition and turbulent) to be filtered in the appropriate manner to give priority the applicable model. An example of the model being applied is graphically illustrated in Figure 4.

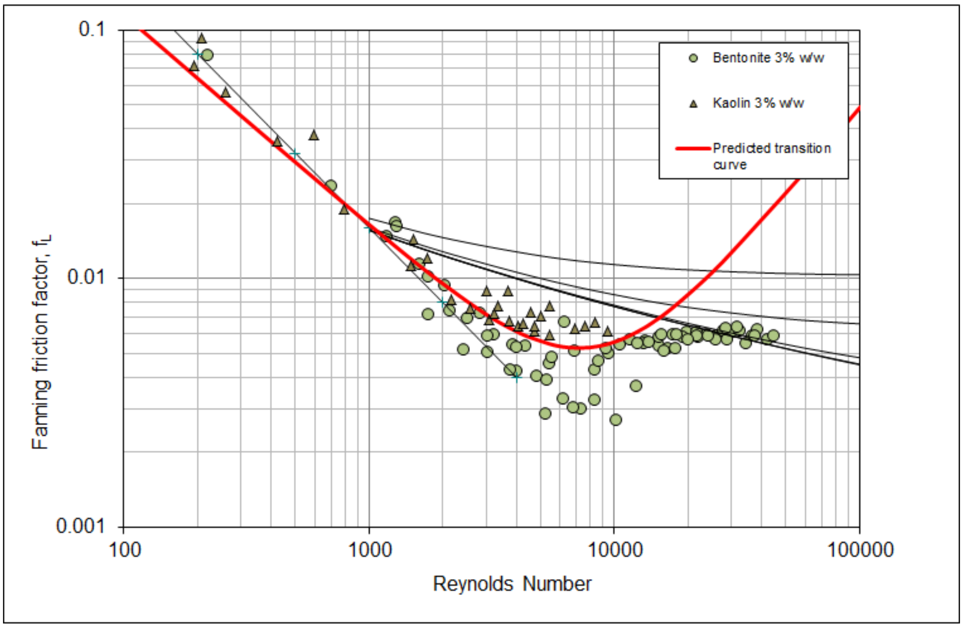

Figure 4. Moody diagram illustrating the model. The solid red line is the predicted transition curve for a fluid with a plastic viscosity of 0.003 Pa.s. Both of the presented experimental sets of data exhibit this particular plastic viscosity.

In Figure 4, open channel data for 3% w/w solutions of Bentonite and Kaolin are plotted. Both exhibited plastic viscosities of about 0.003 Pa.s. Based on this plastic viscosity, a transition curve has been defined by equations 2, 3 and 4. This curve is also plotted in Figure 4 along with the relevant experimental data. Despite some scatter in the data, the model is generally following the data, albeit with some under or over prediction.

4.2 LIMITATIONS OF THE NEW MODEL

The new transition model is only applicable to non-Newtonian fluids with greater viscosity than that of water. The water data of Nikuradse (1932) and the gas data of McKeon et al (2004) are both Newtonian. It was found that the shape and location of the dips in each of these two Newtonian data sets was not in keeping with the general trend observed in the non-Newtonian Haldenwang channel data.

4.3 TESTING THE MODEL

The model has been tested numerically by comparing its absolute prediction error against that of the Hagen-Poiseuille and Colebrook-White equations being applied in the intersection method, as described by Fitton (2008). That method adopts the prediction of the Colebrook-White equation whenever the Hagen-Poisueille equation calculates a friction factor that is smaller for a given Reynolds number (and conversely the Hagen-Poiseuille predictions are adopted whenever the Colebrook-White equation predicts a smaller number for a given Reynolds number). The instances where the new model predicts a friction factor value that is closer to the observed values have been tallied up, as well as the instances where is predicts an figure with a greater absolute error than the intersection method.

In summary, it was found that the new model deviated from the intersection method for 312 of the 798 points of data, of which 64% were improvements in comparison to the intersection method predictions.

4. DISCUSSION AND CONCLUSIONS

A new empirical model for the prediction of the Fanning friction factor for non-Newtonian fluids in the transition regime has been presented. This model does not add to the current theoretical understanding of transition, but instead provides a practical tool for designers and engineers, and a basis for further development.

The new transition model has been found to make some improvement beyond the commonly used intersection method, but it can be seen that there is room for further improvement, particularly as the model makes no attempt to consider any possible physical mechanism(s) that cause the dip in the friction factor when a fluid is flowing in the transition region.

It was previously noted that Slatter and Wasp (2000) and Wilson and Thomas (2006) presented methods for predicting the onset of fully turbulent flow in pipe flow. It is noted here that both of these models featured the yield stress as a key input parameter.

During the empirical curve fitting carried out for this work it was found that the yield stress bore no strong correlations to the dip behavior of any of the data sets.

The model is offered for others to test and validate with other data. It is also hoped that this work can provide a basis for further work to be carried out in the pursuit of a better method of modelling the laminar/turbulent transition. An obvious goal would be a more accurate transition model that also covers Newtonian fluids.

REFERENCES

1. Cheng, N. S. (2008). Formulas for friction factor in transitional regions, Journal of Hydraulic Engineering, ASCE, 134(9), 1357-1362.
2. Fitton, T.G. (2008) Non-Newtonian Open Channel Flow: a simple method of estimation of laminar/turbulent transition and flow resistance", paper presented to Paste 2008, Kasane, Botswana, 6-9 May, 2008.
3. Haldenwang, R (2003) Flow of non-Newtonian fluids in open channels, D.Tech. thesis, Cape Technikon, Cape Town.
4. Haldenwang, R., Slatter, P.T., Vanyaza, S. and Chhabra, R.P. (2004) The effect of shape on laminar flow in open channels for non-Newtonian fluids. Proc. 16th Int.Conf. Hydr. Transp., Santiago, Chile, pp. 311-324.
5. Joseph, D.D. and Yang, B.H. (2009) Friction factor correlations for laminar, transition and turbulent flow in smooth pipes, Physica D Nonlinear Phenomena. 07/2010; 239(14).
6. McKeon, B.J., Swanson, C.J., Zaragola, M.V., Donnelly, R.J. & Smits, A.J. (2004) Friction factors for smooth pipe flow, J. Fluid Mech., 511, 41-44.
7. Metzner, A.B. and Reed, J.C. (1955) Flow of non-newtonian fluids—correlation of the laminar, transition, and turbulent-flow regions, AIChE Journal, Volume 1, Issue 4, pages 434–440, December 1955
8. Nikuradse, J. (1932) Laws of turbulent flow in smooth pipes (English translation). NASA TT F-10: 359 (1966).
9. Ryan, N. W., and Johnson, M. M. (1959) Transistion from Laminar to Turbulent Flow in Pipes, A.I.Ch.E. Journal, 5, 433, 1959.
10. Schaschke, C, (1998) Fluid Mechanics – Worked examples for Engineers, IChemE, Rugby, Warwickshire, UK.
11. Slatter, P.T. (2013) Slurry Pumping Masterclass, Perth, November 2003.
12. Slatter, P.T. and Wasp, E.J. (2000) The laminar/turbulent transition in large pipes. Proc. 10t hInt'l Conf. on Transport & Sedimentation of Solid Particles, AUV, Wrocław, Poland, pp. 389-399.
13. White, F. M., Fluid Mechanics, 7th edition, McGraw. Hill, New York, 2011.
14. Wilson, K.C. and Thomas, A.D. (2006) Analytic Model of Laminar-Turbulent Transition for Bingham Plastics, Canadian J Chem Eng, 84, 5, 520–526, October 2006